

## Problem 2.41

[Difficulty: 2]

**2.41** The velocity distribution for laminar flow between parallel plates is given by

$$\frac{u}{u_{\max}} = 1 - \left(\frac{2y}{h}\right)^2$$

where  $h$  is the distance separating the plates and the origin is placed midway between the plates. Consider a flow of water at  $15^\circ\text{C}$  with maximum speed of  $0.05\text{ m/s}$  and  $h = 0.1\text{ mm}$ . Calculate the force on a  $1\text{ m}^2$  section of the lower plate and give its direction.

**Given:** Velocity distribution between parallel plates

**Find:** Force on lower plate

**Solution:**

Basic equations

$$F = \tau_{yx} \cdot A \quad \tau_{yx} = \mu \cdot \frac{du}{dy}$$
$$\frac{du}{dy} = \frac{d}{dy} u_{\max} \cdot \left[ 1 - \left( \frac{2y}{h} \right)^2 \right] = u_{\max} \cdot \left( -\frac{4}{h^2} \right) \cdot 2y = -\frac{8 \cdot u_{\max} \cdot y}{h^2}$$

so

$$\tau_{yx} = -\frac{8 \cdot \mu \cdot u_{\max} \cdot y}{h^2} \quad \text{and} \quad F = -\frac{8 \cdot A \cdot \mu \cdot u_{\max} \cdot y}{h^2}$$

At the lower surface

$$y = -\frac{h}{2} \quad \text{and} \quad h = 0.1 \cdot \text{mm} \quad A = 1 \cdot \text{m}^2$$
$$u_{\max} = 0.05 \cdot \frac{\text{m}}{\text{s}} \quad \mu = 1.14 \times 10^{-3} \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2} \quad (\text{Table A.8})$$

Hence

$$F = -8 \times 1 \cdot \text{m}^2 \times 1.14 \times 10^{-3} \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2} \times 0.05 \cdot \frac{\text{m}}{\text{s}} \times \frac{-0.1}{2} \cdot \text{mm} \times \frac{1 \cdot \text{m}}{1000 \cdot \text{mm}} \times \left( \frac{1}{0.1} \cdot \frac{1}{\text{mm}} \times \frac{1000 \cdot \text{mm}}{1 \cdot \text{m}} \right)^2$$
$$F = 2.28 \cdot \text{N} \quad (\text{to the right})$$